ISBN: 978-981-18-5852-9
WCSE 2022 Spring Event: 2022 9th International Conference on Industrial Engineering and Applications
doi: 10.18178/wcse.2022.04.044

# Hierarchical Bayesian Recommendation Model for Inter-Company Collaboration Using Cross-Industry Questionnaire

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**Abstract.** In recent years, companies have participated in a lot of brand collaboration, which is expected to improve consumer attitudes toward both brands. However, choosing the appropriate collaborator is a very difficult task. Therefore, we propose a hierarchical Bayesian model for recommending a company for collaboration using questionnaire data rating consumer satisfaction with the company. This model allows us to account for uncertainty, bias, and hierarchical relationships in the questionnaire data. Our experiments showed that the model can estimate the level of satisfaction with the companies, and also confirmed the validity of the company recommendations for collaboration.

**Keywords:** Hierarchical Bayesian modelling, Questionnaire, Recommendation system, Mean-variance model

## 1. Introduction

In Japan, brand collaboration has become common, especially in the food and entertainment industries. Hou et al. [1] showed that collaboration between a reputable product and a less reputable product improves consumers' attitudes toward both brands. In addition, the inclusion of opinions on companies from different industries that do not conform to industry norms may lead to innovative ideas. However, collaboration can have a significant impact on brand image, so choosing the appropriate partner company is an important marketing factor. However, society includes many companies in various industries, and it is difficult to select the most suitable collaborators manually. Therefore, it is desirable to be able to automatically estimate the hypothetical marketing effects of collaboration among companies. With this estimation, we can support an important decision-making process in business strategic planning, which is to select highly effective companies as appropriate collaboration partners. However, it has been difficult to deal comprehensively with the marketing effects of a very large group of companies across different industries.

Therefore, we developed a framework for recommending collaboration partners by using questionnaire data on the consumer satisfaction level for companies. This questionnaire was used in surveys by Oricon Inc., which is the most famous Japanese research company. Their satisfaction surveys have a tremendous impact on the decision-making process of Japanese companies and consumers. In addition, the respondents evaluated their satisfaction with a specific company from various viewpoints. By learning from these data, we constructed a scoring model that outputs the satisfaction levels for different companies for a given customer group when the customer group of a company is input. This model enables us to capture the level of satisfaction that the customers of a hypothetical collaboration partner show toward the target company and to recommend a collaboration partner with high marketing effectiveness. The scoring model employs a hierarchical Bayesian approach. Therefore, it is possible to consider the variability of responses that is likely to occur with a small amount of questionnaire data, the hierarchical relationship of questionnaire data, and the response tendency bias of each individual.

## 2. Related works

Rossi et al. [2] proposed a multivariate ordinal probit model, pointing out that treating response scores as continuous values in questionnaire data evaluated on a discrete ordinal scale causes bias in correlation and

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regression analyses. In addition, another work proposed a model to identify the components of brand equity of a product using a hierarchical Bayesian linear model [3]. In that study, a questionnaire was used to investigate how much information about the price and design of a television was associated with its purchase. By analyzing this consumer purchase information with a hierarchical Bayesian model, the nature of the brand that most influences the purchase was estimated, accounting for the influence of each consumer. In addition, it is known that the hierarchical Bayesian model can account for variation in the number of responses, which may be a problem when analyzing data by industry [4].

It has been suggested that questionnaire data contain a variety of psychological biases that are not assumed by the questioner [5-6]. Specifically, respondents answer based on their implicit standards, and their responses are influenced by time, place, and physical condition. These biases may lead to erroneous results in subsequent analyses, and it is necessary to eliminate them as much as possible.

Thus, while questionnaire data has been analysed for various purposes, none have been used for inter-company collaboration recommendations. In addition, the hierarchical Bayesian model needs to be created according to the hierarchical structure of the questionnaire data. On the other hand, the questionnaire data of Orion Corporation, which was used in this research, was obtained from a wider range of industries than normal questionnaire data, so it was necessary to model according to the unique hierarchical structure.

Therefore, this paper proposes a method to implement intercompany collaboration using an analysis tailored to the unique hierarchical structure.

# 3. Proposed Method

Collaboration is expected to bring in the customers of the collaborative partner as new customers. Therefore, it is desirable to collaborate with a company that has a customer base that highly regards the target company. For this purpose, it is necessary to construct a model that predicts the score for the target company when the customer information of the collaborator is input. This model is called a scoring model in this paper.

In this section, we first explain the characteristics of the data used in this study, namely, the Oricon questionnaire data, and then describe the requirements for the scoring model. Finally, we propose a specific scoring model corresponding to the requirements.

## 3.1. Characters of Oricon Questionnaire Data

The following two questions were common to the entire questionnaire:

- Satisfaction by item
- Importance by item

Satisfaction by item is defined as “How satisfied are you with [respondent’s company name]? Please indicate your level of satisfaction on a 10-point scale for each of the following items.” Importance by item is defined as “What did you consider important when choosing (before choosing) [respondent’s name]?” Here, the available responses for satisfaction by item and the options for importance by item are the same. In addition, the questions are grouped. For example, for theme parks, the three questions “ease of purchasing tickets”, “variety of ticket price plans”, and “acceptability of ticket prices for services” are grouped in terms of “ease of purchasing tickets”. Thus, the questions in a group have similar content, which is determined by Oricon when creating the questions. Furthermore, the mix of questions in question groups differ by industry. In summary, the questionnaire data are based on (1) overall experience, (2) industry, (3) question group, and (4) question. In this paper, we consider these as four hierarchical structures.

## 3.2. Requirements for Scoring Models

To construct a scoring model, three requirements should be considered. The first is to deal with small amounts of data. In this questionnaire, respondents answer for one company that satisfies the conditions among the companies that provide the service. Therefore, the number of respondents varies greatly depending on the company and industry, and many companies did not receive a sufficient number of responses. The second requirement to be considered is the use of the hierarchical structure. As described in section 3.1, the questionnaire data have a hierarchical structure, and some trends depend on the hierarchy. For example, in the hierarchy of question groups, there is a tendency that depends on each question group, such as “ease of buying tickets is difficult to evaluate for all companies." In the hierarchy of industries, there is a tendency common to all question groups in each industry, such as "theme parks are likely to be evaluated highly." In the hierarchy of the overall response, there is a tendency that is common to the whole questionnaire, such as "7 points tends to be the average of the responses in the questionnaire data." By considering such a hierarchical structure, the accuracy of the model may be improved in a situation where the amount of data is limited. The third is to deal with the response tendency. Here, the response tendency refers to the psychological bias Here, the response tendency refers to the psychological bias that each respondent has, as described in section 2. For example, if the response tendency is not corrected, the analysis may not capture the true level of satisfaction because a score of 6 points might be assigned by a person who considers 5 points as the standard, while another person might consider 7 points as the standard and also assign 6 points. Although both respondents are applying different subjective evaluations, and providing different evaluations, both scores of 6 points are treated in the same way. Based on the above, we decided to consider three factors in the scoring model: (1) small amount of data, (2) hierarchical structure, and (3) response tendency.

## 3.3. Scoring Model

1) Overview: Let $y_{c,f,i} \in \{1,2,\cdots,10\}$ denote the targeting company c. Then, their mean $\bar{y}_{c,f}$ can be calculated satisfaction level given by individual I to question group f as follows.

$$\bar{y}_{c,f} \coloneqq \frac{1}{|f|}\sum_{i\in I} y_{c,f,i} \tag{1}$$

Then, we consider the following equation as a method to calculate the score of company c from all questionnaire responses targeting that company.

$$\boldsymbol{c} \coloneqq \sum_{\boldsymbol{i}\in \boldsymbol{I}} \bar{\boldsymbol{y}}_{\boldsymbol{c},\boldsymbol{f}} \times \bar{\boldsymbol{w}}_{\boldsymbol{f}} \tag{2}$$

However, $\bar{w}_f$ is the average of the importance of each item for the questionnaire respondents providing input for question group $\boldsymbol{f}$. The correlation coefficients were 0.998 for the theme park industry and 0.996 for cafes. In this study, the scores are calculated based on Oricon's total score. Therefore, the scores are estimated based on equations (1) and (2).

2) Questionnaire response regression model: For individuals who have not responded to the questionnaire, it is necessary to estimate their satisfaction level. For this reason, we explain the method of estimating the satisfaction level based on the questionnaire response regression model. The following regression model is fit to the satisfaction.

$$y_{c,f,i} \in \{1, 2, \cdot\cdot\cdot, 10\}:$$

$$y_{c,f,i} = \theta_{c,f} + F_{c,f}(x_i) + b_i + \epsilon. \tag{3}$$

where $\theta_{c,f}$ is the base satisfaction for company $c$ with question group $F_{c,f}(\boldsymbol{x}_i)$ is a correction term that depends on demographic attributes (such as gender and age), $\boldsymbol{x}_i$ is error term. For the demographic attribute term $F_{c,f}(\boldsymbol{x}_i)$, we the response tendency bias of individual $\boldsymbol{i}$, $b_i$, and $\epsilon$ is the propose two patterns: a linear model and a neural network (NN) model. The linear model is one in which $F_{c,f}(\boldsymbol{x}_i) = \boldsymbol{v}_{c,f}^T \boldsymbol{x}_i$. The NN model simply supplements $F_{c,f}(\boldsymbol{x}_i)$ with a fully connected NN. The linear model is highly interpretable, while the NN model is highly expressive, so it is best to use them separately depending on the case.

To train the questionnaire response regression model (3), we performed Bayesian estimation to account for the variability caused by the small amount of data, which is the first requirement. This allowed us to obtain a posterior predictive distribution of satisfaction. In addition, we can know not only the expected value but also the variability of the score for a company, and we can express the size of the credit interval due to the small number of respondents.

Furthermore, we considered the second requirement, namely, the use of a hierarchical structure. A hierarchical Bayesian estimation model was used when information was available at several different levels of observation units. For example, in the hierarchy of question groups, the satisfaction level of "ease of buying tickets" for theme park A was set to follow the distribution of "ease of buying tickets" for all theme park companies. In the next hierarchy of industries, we made the various question sets follow the distribution for all theme parks to facilitate achieving similar estimation results. Finally, by setting the entire cross-industry hierarchy to arise from the same distribution, we can obtain distributions similar to those of other industries, even for industries for which we lack information. For the estimation of the base satisfaction $\theta_{c,f}$, we introduced three hierarchies: question group, industry, and total. In the linear model of the demographic attribute term $F_{c,f}(\boldsymbol{x}_i) = \boldsymbol{v}_{c,f}^T \boldsymbol{x}_i$ we introduced a similar hierarchy for the weights $\boldsymbol{v}_{c,f}$ for each attribute. In contrast, we introduced only the hierarchies for industry and overall because the individual response tendency bias $b_i$ seems to be independent of the question group.

Given the above considerations, namely, a hierarchical Bayesian linear model with a linear model of the demographic attribute terms, we obtained the overall structure shown in Figure 1. The y in the center of the figure represents the questionnaire responses, and each of the other circles is a parameter. It is assumed that μ, θ, v, and b are normally distributed, σ follows the Half-Cauchy distribution, and τ follows the inverse gamma distribution as prior distributions. Parameters D and X are the number of industries and the number of demographic attributes (dimensions of $v_{c,f}$), respectively, and $C_d$, $F_d$, and $I_d$ are the number of companies, groups of questions, and individuals, respectively, when we focus on industries $d = 1, 2, \cdots, D$.

In the hierarchical Bayesian NN model, in which the demographic attribute term is used as an NN model, the part related to v on the right side of Fig.1 is supplemented by the NN. In this study, we adopted a fully coupled model in which both the embedding layer and the middle layer consist of 3 layers of 32 dimensions as the structure of the network.

3) Score calculation: In the questionnaire response regression model (3), to account for the psychological bias of individuals, a bias term $b_i$ representing the response tendency of individuals is introduced separately from the term $F_{c,f}(x_i)$ that depends on the demographic attributes of individuals. Equation (4), which excludes the response tendency bias term and the error term, is then used to the estimate the estimated value of satisfaction $\tilde{y}_{c,f,i}$ without effect of bias.

$$\tilde{y}_{c,f,i} \coloneqq \theta_{c,f} + F_{c,f}(\boldsymbol{x}_i) \tag{4}$$

This makes it possible to eliminate the third requirement, namely, the response tendency bias. In summary, after training the questionnaire response regression model, the distribution of the score of company $c$ indicated by a particular customer segment $I$, such as respondents of the hypothetical collaborator company, can be calculated by the following procedure:

1) Compute the posterior predictive distribution of satisfaction $y_{c,f,i}$ from equation (4) for $i \in I$
2) Calculate the posterior predictive distribution of $y_{c,f}$ with equation (1)
3) Calculate the distribution of the score of company c with equation (2).

The $w_f$ value required for the calculation of equation (2) depends only on the question group. Here, we use the value obtained by Bayesian estimation of the item-specific importance of questionnaire respondents to question group $\boldsymbol{f}$ and assume that it follows a Bernoulli distribution.

# 4. Verification Using Actual Survey Data

## 4.1. Data and Implementation Overview

The data used in this paper were specially prepared by Oricon Inc., for the 2020 Data Analysis Competition. Only companies with more than 100 respondents were included in the survey, and the number of respondents and companies used in the analysis are shown in Table 1.

We used five demographic attributes for each individual: gender, age, region of residence, annual income, and occupation. The Bayesian inference was implemented using NumPyro [7], a Bayesian modelling package for Python. To estimate the posterior distribution, we used the stochastic variational inference (SVI) Bayesian method [8] because of its limited runtime, and optimized the parameters using Adam [9] with a learning rate of 0.002 and 30,000 iterations. For the assumption of the posterior distribution required by SVI, we chose a normal distribution, in which the variance–covariance matrix is diagonal. In the preliminary experiments, the parameters of the prior distribution were varied and the shape of the distribution was changed, but it was confirmed that the obtained posterior distribution was robust. A hierarchical Bayesian linear model was used to calculate the scores of 15 companies in the theme park industry using the questionnaire responses for each company as input. The results are shown in Figure 2. The model confirmed that the distribution of company scores can be estimated.

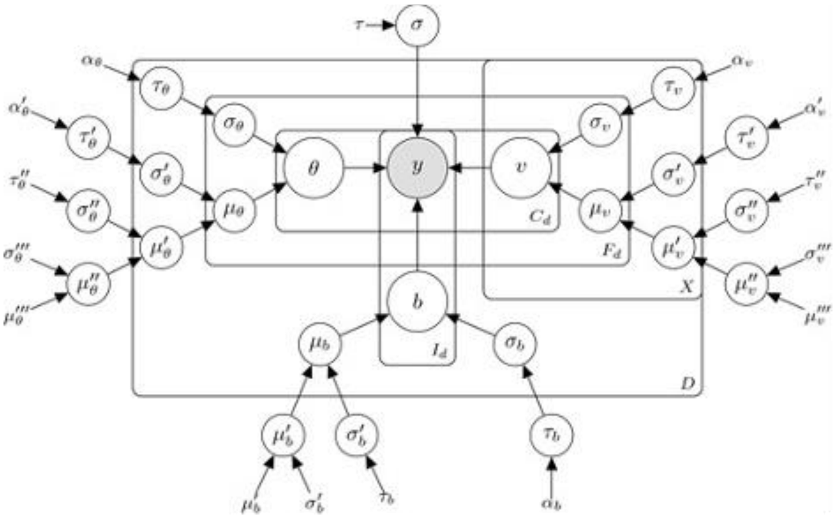

Fig 1: Graphical representation of the Bayesian linear model.

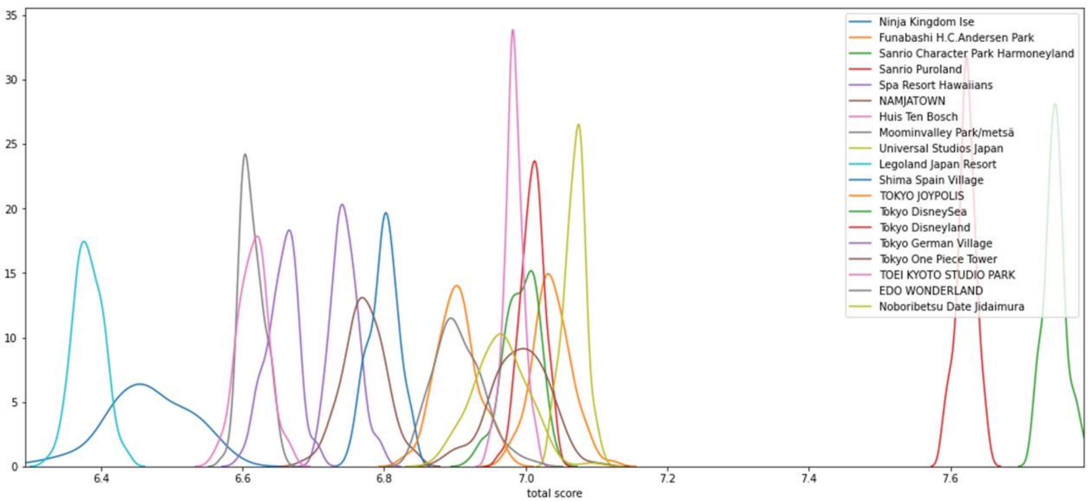


Fig 2: Estimation results of the hierarchical Bayesian linear model.

| Type of industry | Number of respondents | Number of companies |
|---|---|---|
| Theme Park | 5454 | 15 |
| Cafe | 9001 | 15 |
| Drugstore | 8381 | 19 |
| Hotel | 15,245 | 38 |
| eBook | 5642 | 8 |
| Home appliances | 7442 | 10 |
| Movie theater | 5037 | 11 |
| Total | 56,202 | 116 |

Table 1: Data used

## 4.2. Generalization Error of Survey Response Regression Model

The results of the calculations are shown in Table 2. It can be seen that the Watanabe–Akaike information criterion [10] improved in the order of the base model, the response tendency consideration model, and the two proposed models. Therefore, it can be said that the proposed model is a regression model that is more appropriate than the simple method for these data. Here, the hierarchical Bayesian linear model and the hierarchical Bayesian NN model slightly outperform the hierarchical Bayesian NN model, although the hierarchical Bayesian linear model is superior in that it can directly confirm the effect of each demographic attribute. However, the hierarchical Bayesian linear model is superior to the hierarchical Bayesian NN model in that the effect of individual demographic attributes can be directly observed. The difference in accuracy may vary depending on the demographic attributes used and the amount of data.

### 4.3. Discussion of Recommendation Results

To qualitatively evaluate the appropriateness of the proposed model for recommending collaboration, the customer groups with the highest scores were selected for Tokyo Disneyland and 109 Cinemas.

We extracted the top five companies in each industry (excluding those in the same industry) that had xxx. The scores were calculated using a hierarchical Bayesian linear model, and the expected values were aggregated. The results for Tokyo Disneyland are shown in Table 4. It can be seen that companies that have already collaborated with Tokyo Disneyland, such as Mitsui Garden Hotels, UCC Cafe Mercado, and Starbucks Coffee, were recommended. In addition, companies located in the vicinity of Tokyo Disneyland, such as Mitsui Garden Hotels, Weird Hotel, and AEON Cinemas, are also in the top rank, and their recommendations are highly appropriate. The results for 109 Cinemas are shown in Table 4. The results for 109 Cinemas are consistent with the results for Tokyo Disneyland, as Tokyo Disneyland ranked second to 109 Cinemas in terms of movie theaters.

| Model | Gender | Age | Region | Annual income | Occupation | Average |
|---|---|---|---|---|---|---|
| Base model | 2.277 | 2.271 | 2.278 | 2.382 | 2.302 | 2.275 |
| Response tendency analysis model | 1.798 | 1.780 | 1.789 | 1.913 | 1.840 | 1.795 |
| Hierarchical Bayesian linear model | 1.777 | 1.757 | 1.762 | 1.884 | 1.802 | 1.774 |
| Hierarchical Bayesian NN model | 1.776 | 1.755 | 1.761 | 1.882 | 1.799 | 1.773 |

Table 2: Generalization error for each model

| Rank | Cafes | Drugstores | Hotels | Home appliances | Movie theaters | eBook services |
|---|---|---|---|---|---|---|
| 1st | Starbucks Coffee | Tsuruha Drug | Daiwa Roynet Hotels | Hotel Joshin | Sapporo Cinema Frontier | Niko Niko Manga |
| 2nd | Musashinomori Coffee | Sapporo Drugstore | Richmond Hotels | K's Denki | 109 Cinemas | Mecha Comics |
| 3rd | Komeda Coffee Shop | Kusuri no Aoki | ANA Crowne Plaza Hotels | EDION | MOVIX | LINE Manga |
| 4th | Tully's Coffee | Wants | Mitsui Garden Hotels | Yodobashi Camera | Corona Cinema World | Renta! |
| 5th | Ueshima Coffee Shop | Create | ANA InterContinental (Tokyo, Yokohama, Osaka) | Bic Camera | United Cinemas | Shonen Jump+ |

Table 3: Recommendation results for Tokyo Disneyland

## 5. Application

In the proposed scoring model, it is also possible to obtain the contemporaneous posterior distribution of the scores for multiple companies. As one of the applications of this contemporaneous posterior distribution, we consider risk diversification using the framework of the portfolio selection problem.

### 5.1. Recommendations for Multiple Companies

In the numerical experiments in the previous section, we assumed a collaboration with one company, but it is possible to collaborate with multiple companies at the same time. In such a case, it may be necessary to consider not only the selection of the collaborating companies but also the scale of the collaboration due to cost constraints. In other words, it is necessary to construct a portfolio that considers the scale (ratio) of collaboration. The problem of constructing a portfolio with small risks while satisfying certain conditions is known as the portfolio selection problem, and it has been applied in fields other than finance [11]. In this paper, we use the framework of the portfolio selection problem to control the risk in the collaboration with multiple companies.

### 5.2. Formularization

In this study, we applied the mean-variance model [11], which is a typical portfolio selection problem. In other words, we set the minimum score that we want to obtain and find the collaboration ratio that minimizes the variance of the score among the candidate companies. As already mentioned, the proposed scoring model provides the distribution of the scores, so it is easy to obtain their average. In addition, the variance-covariance matrices among the scores can also be calculated because the simultaneous posterior distributions of multiple scores can be obtained. By using this approach, we can construct a standard mean and variance model. In addition, we assume a realistic situation ad set upper and lower bounds to remove cases where the ratio is extremely low or high. We also set the maximum number of companies that can collaborate and the budget for collaboration. The above problem can be formulated as the following mixed-integer quadratic programming problem with the collaboration ratio for each company as a variable:

| Rank | Cafes | Drugstores | Hotels | Home appliances | Movie theaters | eBook services |
|---|---|---|---|---|---|---|
| 1st | Tokyo DisneySea | Starbucks Coffee | Discount Drug Cosmos | Daiwa Roynet Hotels | Yodobashi Camera | Renta! |
| 2nd | Tokyo Disneyland | Musashinomori Coffee | Sapporo Drugstore | Richmond Hotels | Bic Camera | Mecha Comics |
| 3rd | Universal Studios Japan | Komeda Coffee Shop | Tsuruha Drug | Mitsui Garden Hotels | EDION | Niko Niko Manga |
| 4th | Funabashi Andersen Park | Tully's Coffee | ANA InterContinental (Tokyo, Yokohama, Osaka) | Kusuri no Aoki | JOSIN | LINE Manga |
| 5th | Sanrio Puroland | Ueshima Coffee Shop | ANA Crowne Plaza Hotels | Wants | K's Denki | eBookJapan |

Table 4: Recommendation results for Tokyo Disneyland

| Collaboration company | Ratio |
|---|---|
| Hotel Root Inn | 55% |
| DisneySea | 35% |
| Kawati | 10% |

Table 5: Distribution to reduce score variability

| Collaboration company | Ratio |
|---|---|
| DisneySea | 77% |
| Mitsui Garden Hotels | 13% |
| Discount Drug Cosmos | 10% |

Table 6: Distribution to increase score expectations

$$\begin{aligned} \min \quad & \mathbf{x}^T \mathbf{V} \mathbf{x} \\ \text{s.t.} \quad & \mathbf{x}^T \boldsymbol{\mu} \geq E_{min} \\ & \textstyle\sum_i y_i \leq N_{company}, \\ & l_i y_i \leq x_i \leq u_i y_i \quad \forall i, \\ & \textstyle\sum_{i \in D_k} y_i \leq d \quad \forall D_k \in D, \\ & \textstyle\sum_i x_i c_i^c + \sum_i y_i c_i^d \leq C, \\ & \textstyle\sum_i x_i \leq 1, \\ & y_i \in \{0,1\} \ \forall i, \\ & x_i \in [0,1] \ \forall I, \end{aligned} \tag{5}$$

where $x_i$ is the collaboration fraction of company **i** and $y_i$ is a binary variable representing whether company i collaborates with another company i. The constants are as follows:

- **V**: Variance-covariance matrix between scores
- **μ**: Average vector of scores
- $E_{min}$: Minimum collaboration effect required
- $N_{company}$: Maximum number of collaborators
- $l_i$: Minimum percentage allocated to company **i**
- $u_i$: Maximum percentage allocated to company **i**

- d: Maximum number of companies for collaboration within the same industry
- D: Set of companies belonging to each industry
- $c_i^d$: Fixed cost of collaborating with company i
- $c_i^c$: Variable cost coefficient for collaborating with company i
- C: Budget.

### 5.3. Consideration of the Results of Multiple Company Recommendations

The mixed-integer quadratic programming problem (5) can be solved using Gurobi [12], a general-purpose solver. In the optimization calculation, the constants other than the mean vector of scores $\mu$ and the variance-covariance matrix $V$ need to be set realistically. The optimal distribution ratio when $E_{\min}$ is set low and high is summarized in Table 5 and Table 6, respectively. When $E_{\min}$ is set low and the priority is to reduce risk (Table 5), the distribution ratios are relatively equal. In contrast, when $E_{\min}$ is high and the priority is to increase the collaboration effect (Table 6), the distribution weight is large for DisneySea. Thus, optimization using the framework of the portfolio selection problem enables us to control the variation of the collaboration effect and to make recommendations that meet the needs of companies.

## 6. Conclusion

In this paper, we propose a framework for recommending companies for collaboration. The framework considers the following three points:

1) Variation due to a small number of respondents
2) Survey-specific hierarchical structure
3) Individual response tendency bias.

In addition, the effectiveness of the proposed framework was verified using actual questionnaire data. The results confirmed that survey data are suitable as model input and that the recommendation results are valid. Finally, as an example of further application, we proposed a method to make collaborative recommendations for multiple companies.

Three issues are raised as future challenges. First, to establish a collaboration, it is necessary to consider the convenience for and benefit to all companies involved. In the future, we would like to consider collaborations that are beneficial to both companies. Second, in this study, we did not consider the overlap of customers among the collaborating companies, but in reality, the effect of collab- oration is considered to be weak for companies with many overlapping customers. Therefore, when there is information about the overlap of customers, it is necessary to propose a model that avoids such overlap and proposes a different collaboration partner. Finally, although we considered collaboration among companies, this model can be applied to collaboration among products. Therefore, there is room for empirical research on the model with reduced granularity for inter-product collaboration.

## 7. Acknowledgements

This research was conducted as part of the Data Analysis Competition hosted by the Joint Association Study Group of Management Science. We would like to thank the organizers and the Oricon Corporation for providing us with the data to conduct this research. Thank you very much.

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